

The Extended Helmholtz-Ellis JI Pitch Notation

microtonal accidentals designed by Marc Sabat and Wolfgang von Schweinitz, 2004 (rev. 2018)

Translated Into Color Notation [original pdf is at www.MarcSabat.com/pdfs/fulllegendE.pdf](http://www.MarcSabat.com/pdfs/fulllegendE.pdf)

3-LIMIT (PYTHAGOREAN) INTERVALS

$\flat\flat$ \flat \natural \sharp \times

 $\flat\flat$ \flat \natural \sharp \times

 w is used to cancel y, g, etc.

FUNCTION OF THE ACCIDENTALS

notate 35 pitches from the series of untempered perfect fifths
 $(3/2) \approx \pm 702.0$ cents;
perfect fifth (3/2); perfect fourth (4/3); major wholetone (9/8)
 $w5 = wa$ 5th $w4 = wa$ 4th $w2 = wa$ 2nd

5-LIMIT (PTOLEMAIC) INTERVALS

$y\flat\flat$ $y\flat$ y $y\sharp$ $y\times$ $g\flat\flat$ $g\flat$ g $g\sharp$ $g\times$

 $y\flat\flat$ $y\flat$ y $y\sharp$ $y\times$ $g\flat\flat$ $g\flat$ g $g\sharp$ $g\times$

 $y\flat\flat$ $y\flat$ y $y\sharp$ $y\times$ $g\flat\flat$ $g\flat$ g $g\sharp$ $g\times$

 $y\flat\flat$ $y\flat$ y $y\sharp$ $y\times$ $g\flat\flat$ $g\flat$ g $g\sharp$ $g\times$

 $y\flat\flat$ $y\flat$ y $y\sharp$ $y\times$ $g\flat\flat$ $g\flat$ g $g\sharp$ $g\times$

 $y\flat\flat$ $y\flat$ y $y\sharp$ $y\times$ $g\flat\flat$ $g\flat$ g $g\sharp$ $g\times$

$y3 = yo$ 3rd $g3 = gu$ 3rd $y6 = yo$ 6th $g6 = gu$ 6th
 notate an alteration by one syntonic comma $(81/80) \approx \pm 21.5$ cents;
major third (5/4); minor third (6/5); major sixth (5/3); minor sixth (8/5);
minor wholetone (10/9) $y2 = yo$ 2nd

notate an alteration by two syntonic commas
 $(81/80) \cdot (81/80) \approx \pm 43.0$ cents;
augmented fifth (25/16); diminished fourth (32/25)
 $yy5 = yoyo$ 5th $gg4 = gugu$ 4th

notate an alteration by three syntonic commas
 $(81/80) \cdot (81/80) \cdot (81/80) \approx \pm 64.5$ cents;
minor diesis (128/125)
 $g^3 2 = trigu$ 2nd

7-LIMIT (SEPTIMAL) INTERVALS

z r

 z r

 zz rr

$z7 = zo$ 7th $r2 = ru$ 2nd
 notate an alteration by one septimal comma $(64/63) \approx \pm 27.3$ cents;
natural seventh (7/4); septimal wholetone (8/7); $7/5 = zg5 = zogu$ 5th
septimal diminished fifth (7/5); septimal tritone (10/7); $10/7 = ry4 = ruyo$ 4th
septimal minor third (7/6); septimal quartertone (36/35)
 $z3 = zo$ 3rd $rg1 = rugu$ unison
 notate an alteration by two septimal commas
 $(64/63) \cdot (64/63) \approx \pm 54.5$ cents;
septimal sixthtone (49/48)
 $zz2 = zozo$ 2nd

11-LIMIT (UNDECIMAL) INTERVALS

$1o$ $1u$

notate an alteration by one undecimal quartertone
 $(33/32) \approx \pm 53.3$ cents; $1o4 = ilo$ 4th $1u5 = lu$ 5th
undecimal augmented fourth (11/8); undecimal diminished fifth (16/11)

13-LIMIT (TRIDECIMAL) INTERVALS

$3o$ $3u$

notate an alteration by one tridecimal thirddone $(27/26) \approx \pm 65.3$ cents;
 $3o6 = tho$ 6th $3u3 = thu$ 3rd
tridecimal neutral sixth (13/8); tridecimal neutral third (16/13)

PRIMES IN THE HARMONIC SERIES OCTAVE 16 - 32 (5-limit signs are given here relative to "A")

$17o\flat$ $17u\sharp$

 $19o$ $19u$

notate an alteration of the 5-limit accidental by one 17-limit schisma
 $(16/17) \cdot (16/15) = (256/255) \approx \pm 6.8$ cents;
Galileo's "equal-tempered" semitone (18/17); $17u1 = su$ semitone
17-limit diminished seventh chord 10:12:14:17 $g, 17og7(zg5)$ chord

notate an alteration by one 19-limit schisma
 $(19/16) \cdot (27/32) = (513/512) \approx \pm 3.4$ cents;
19-limit minor third (19/16); 19-limit minor triad 16:19:24
 $19o3 = ino$ 3rd $C19o = C$ ino

$\uparrow\sharp$
23o \sharp

$\downarrow\flat$
23u \flat

notate an alteration by one 23-limit comma
 $(23/16) \cdot (8/9) \cdot (8/9) \cdot (8/9) \approx \pm 16.5$ cents;
raised leading tone (23/12) 23o7 = twenty-tho 7th

$\equiv\sharp$ or $\uparrow\sharp$
29o

$\equiv\flat$ or $\downarrow\flat$
29u

notate an alteration of the 5-limit accidental by one 29-limit comma
 $(29/16) \cdot (5/9) = (145/144) \approx \pm 12.0$ cents
29/16 = 29o7 = twenty-no 7th

$-d$
31o \flat

$+t$
31u \sharp

notate an alteration of the 11-limit accidental by one 31-limit schisma
 $(32/31) \cdot (32/33) = (1024/1023) \approx \pm 1.7$ cents
31/16 = 31o7 = thirty-wo 7th

PRIMES IN THE HARMONIC SERIES OCTAVE 32 - 64 (5-limit signs are given here relative to "A")

$\{\}\uparrow\sharp$
37o \sharp

$\{\}\downarrow\flat$
37u \flat

notate an alteration of the 11-limit accidental by one 37-limit schisma
 $(36/37) \cdot (33/32) = (297/296) \approx \pm 5.8$ cents
37/32 = 37o2 = thirty-so 2nd

$\{\#\}$
41o \sharp

$\{\flat\}$
41u \flat

notate an alteration of the 5-limit accidental by one 41-limit schisma
 $(32/41) \cdot (81/64) \cdot (81/80) = (6561/6560) \approx \pm 0.3$ cents
41/32 = 41o3 = forty-wo 3rd

$\{\uparrow\}$
43o

$\{\downarrow\}$
43u

notate an alteration by one 43-limit comma
 $(43/32) \cdot (3/4) = (129/128) \approx \pm 13.5$ cents
43/32 = 43o4 = forty-tho 4th

$\{\#\}$ or $\{\#\#\}$
47o \sharp

$\{\flat\}$ or $\{\flat\flat\}$
47u \flat

notate an alteration of the 7-limit accidental by one 47-limit schisma
 $(32/47) \cdot (48/49) \cdot (3/2) = (2304/2303) \approx \pm 0.8$ cents
47/32 = 47o4 = forty-so 4th

$\{\#\#\}$
53o \sharp

$\{\#\flat\}$
53u \flat

notate an alteration of the 5-limit accidental by one 53-limit comma
 $(32/53) \cdot (5/3) = (160/159) \approx \pm 10.9$ cents
53/32 = 53o4 = fifty-tho 6th

$\{\#\#\}$
59o \sharp

$\{\#\flat\}$
59u \flat

notate an alteration of the 13-limit accidental by one 59-limit schisma
 $(32/59) \cdot (24/13) = (768/767) \approx \pm 2.3$ cents
59/32 = 59o7 = fifty-no 7th

$\{\#\#\}$
61o \sharp

$\{\#\flat\}$
61u \flat

notate an alteration of the 7-limit accidental by one 61-limit schisma
 $(61/32) \cdot (21/40) = (1281/1280) \approx \pm 1.4$ cents
61/32 = 61o7 = sixty-wo 7th

IRRATIONAL AND TEMPERED INTERVALS

$\flat\flat$ \flat \flat \sharp \times
 $\wedge\flat\flat$ $\wedge\flat$ \wedge $\wedge\sharp$ $\wedge\times$
 $\vee\flat\flat$ $\vee\flat$ \vee $\vee\sharp$ $\vee\times$

notate the respective Equal Tempered Semitone;
may be combined with a cents indication to notate any pitch

$\wedge 5$ = up 5th = the edo's best 5th raised by one edostep

$\vee 5$ = down 5th, $\vee\vee 5$ = double-down 5th, etc.

NOTE ABOUT CENTS INDICATIONS

optional cents indications may be placed above or below the respective accidentals and are always understood in reference to Equal Tempered semitones, as implied by the Pythagorean accidentals if the cents exceed ± 50 the closest pitch as indicated on a tuner may be written as text, e.g. F#-35

TEXT NOTATION

in addition to the accidentals, a useful text shorthand for musicians combines the prime constituents of a ratio with the symbols u and o to indicate harmonic space coordinates: for example 7° or $u11$

u o z $1u$

FONT

The HEJISMuFL font used here (2018) is freely available for download from www.plainsound.org

* special thanks to Juhani Nuorvala for suggesting use of a distinct alternate symbol for 29°